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Sampled-Data Control of Nonlinear Retarded Systems

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Outline

- Nonlinear Retarded Systems, Preliminary Notions and Problem Statement
- Control Lyapunov-Krasovskii Functionals and Steepest Descent State Feedbacks
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- Linearizers and Virtual Stabilizers as Stabilizers in the Sample-and-Hold Sense
- Work in Progress and Future Developments

Stabilization in the Sample-and-Hold Sense

F.H. Clarke, Y.S. Ledyaev, E.D. Sontag, A.I. Subbotin, "Asymptotic controllability implies feedback stabilization," *IEEE Transactions on Automatic Control*, Vol. 42, pp. 1394-1407, 1997.

F.H. Clarke, "Discontinuous Feedback and Nonlinear Systems", Plenary Lecture at IFAC Conference on Nonlinear Control Systems (NOLCOS), Bologna, Italy, 2010, IFAC-PapersOnline.

Systems Described by RFDEs

$$\dot{x}(t) = f(x_t, u(t)), \qquad t \geq 0, \; a.e.,$$
$$x(\tau) = x_0(\tau), \qquad \tau \in [-\Delta, 0], \quad x_0 \in \mathcal{C}, \tag{1}$$

$x(t) \in R^n$, n is a positive integer; Δ is a positive integer, the maximum involved time-delay; $\mathcal{C}$ is the Banach space of continuous functions mapping $[-\Delta, 0]$ to R^n, endowed with the norm of uniform topology, denoted with $\|\cdot\|_\infty$; $x_t \in \mathcal{C}$ is defined as $x_t(\tau) = x(t+\tau)$, $\tau \in [-\Delta, 0]$; f is a map from $\mathcal{C} \times R^m$ to R^n, Lipschitz on bounded sets, zero at zero; m is a positive integer; $u(t) \in R^m$ is a Lebesgue measurable, locally essentially bounded signal.

For a positive real r, $\mathcal{C}_r = \{\phi \in \mathcal{C} : \|\phi\|_\infty \leq r\}$

Definition 1. *Let* $V : \mathcal{C} \to R^{+}$ *be a locally Lipschitz functional. The derivative* $D^{+}V : \mathcal{C} \times R^{m} \to R^{\star}$ *of the functional* V *is defined, in the Driver's form (see Driver, 1962, Burton, 1985, P. & Jiang, 2006, Karafyllis, 2006), for* $\phi \in \mathcal{C}$, $v \in R^{m}$, *as follows*

$$D^{+}V(\phi, v) = \limsup_{h \to 0^{+}} \frac{1}{h}\Big(V\left(\phi_{h,v}\right) - V(\phi)\Big),$$

where $\phi_{h,v} \in \mathcal{C}$ *is given by*

$$\phi_{h,v}(s) = \begin{cases} \phi(s+h), & s \in [-\Delta, -h], \\ \phi(0) + f(\phi, v)(h+s), & s \in (-h, 0] \end{cases}$$

Theorem 2. *(Karafyllis, P., Jiang, EJC 2008) Let in the RFDE (1)* $u(t) = 0$, $t \geq 0$. *The system described by the RFDE (1) is* $0-$*GAS if and only if there exist a locally Lipschitz functional* $V : \mathcal{C} \to R^+$ *and functions* α_1, α_2 *of class* $\mathcal{K}_\infty$, α_3 *of class* $\mathcal{K}$, *such that,* $\forall \phi \in \mathcal{C}$, *the following inequalities hold:*

i) $\alpha_1(\|\phi\|_\infty) \leq V(\phi) \leq \alpha_2(\|\phi\|_\infty)$;

ii) $D^+V(\phi, 0) \leq -a_3(\|\phi\|_\infty)$

Definition 3. *(P., SICON 2014) A functional* $V : \mathcal{C} \to R^+$ *is said to be smoothly-separable if there exist a function* $V_1 \in C^1_L(R^n; R^+)$*, a locally Lipschitz functional* $V_2 : \mathcal{C} \to R^+$*, functions* β_i *of class* $\mathcal{K}_\infty$*,* $i = 1, 2$*, such that, for any* $\phi \in \mathcal{C}$*, the following equality/inequalities hold*

$$V(\phi) = V_1(\phi(0)) + V_2(\phi), \qquad \beta_1(|\phi(0)|) \leq V_1(\phi(0)) \leq \beta_2(|\phi(0)|)$$

Definition 4. *(Artstein, NA 1983, Jankovic, TAC 2001, P., SICON 2014) A smoothly-separable functional $V : \mathcal{C} \to R^+$ is said to be a CLKF if there exist functions γ_1, γ_2 of class $\mathcal{K}_\infty$ such that the following inequalities hold*

i) $\gamma_1(|\phi(0)|) \leq V(\phi) \leq \gamma_2(\|\phi\|_\infty), \ \forall \phi \in \mathcal{C}$;

ii) $\inf_{u \in R^m} D^+ V(\phi, u) < 0, \ \forall \phi \in \mathcal{C}, \ \phi(0) \neq 0.$

Definition 5. *(P., SICON 2014) A map $k : \mathcal{C} \to U$ (continuous or not) is said to be a steepest descent feedback, induced by a CLKF V, if the following condition holds: there exist $m \in \{0, 1\}$, positive reals η and μ, a function $p \in C^1_L(R^+; R^+)$, of class $\mathcal{K}_\infty$, such that, $\forall \phi \in \mathcal{C}$,*

$$mD^+V(\phi, k(\phi)) + \eta \max\{0, D^+p \circ V_1(\phi, k(\phi)) + \mu p \circ V_1(\phi(0))\} \leq 0$$

Recall: $V(\phi) = V_1(\phi(0)) + V_2(\phi)$

$$\dot{x}(t) = x(t-\Delta) + |x(t)|u(t)$$

$$V(\phi) = V_1(\phi(0)) + V_2(\phi),\ \phi \in \mathcal{C}$$
$$V_1(x) = x^2,\ x \in R, \qquad V_2(\phi) = \int_{-\Delta}^{0} 2\phi^2(\tau)d\tau,\ \phi \in \mathcal{C}$$
$$k(\phi) = -2sgn(\phi(0))$$

V is CLKF, k is a (discontinuous) steepest descent feedback. Indeed, for $m = 1$, $\eta = 0.1$, $p = I_d$, $\mu = 1$, we have, for any $\phi \in \mathcal{C}$:

$$\inf_{u \in R} D^+V(\phi, u) \leq D^+V(\phi, k(\phi)) \leq -\phi^2(0) - \phi^2(-\Delta),$$

$$mD^+V(\phi, k(\phi)) + \eta \max\{0, D^+p \circ V_1(\phi, k(\phi)) + \mu V_1(\phi(0))\} \leq$$
$$-\phi^2(0) - \phi^2(-\Delta) + 0.1 \max\{0, -2\phi^2(0) + \phi^2(-\Delta)\} \leq 0$$

Assumption 6. *There exists a positive real* q *such that the initial condition* $x_0 \in W^{1,\infty}$, *and* $ess\,\sup_{\theta\in[-\Delta 0]}\left|\frac{dx_0(\theta)}{d\theta}\right| \leq q$. *There exist a CLKF* V *and an induced steepest descent feedback* k *(continuous or not). The map* $\phi \to D^+V_2(\phi, u)$ *is Lipschitz on bounded subsets of* $\mathcal{C} \times R^m$.

Definition 7. *(Clarke et al., TAC 1997, P., SICON 2014)* *A partition* $\pi = \{t_i,\ i = 0, 1, \ldots\}$ *of* $[0, +\infty)$ *is a countable, strictly increasing sequence* t_i*, with* $t_0 = 0$*, such that* $t_i \to +\infty$ *as* $i \to +\infty$*. The diameter of* π*, denoted* $diam(\pi)$*, is defined as* $\sup_{i \geq 0} t_{i+1} - t_i$*. The dwell-time of* π*, denoted* $dwell(\pi)$*, is defined as* $\inf_{i \geq 0} t_{i+1} - t_i$*. For any positive reals* $a \in (0, 1]$*,* $b > 0$*,* $\pi_{a,b}$ *is any partition* π *with* $ab \leq dwell(\pi) \leq diam(\pi) \leq b$*.*

Definition 8. *(Clarke et al., TAC 1997, P., SICON 2014) We say that a feedback $F : \mathcal{C} \to R^m$ (continuous or not) stabilizes the system described by the RFDE in the sample-and-hold sense if, for every positive reals r, R, $0 < r < R$, $a \in (0, 1]$, there exist a positive real δ depending upon r, R, q and Δ, a positive real T, depending upon r, R, q, Δ and a, and a positive real E, depending upon R and Δ, such that, for any partition $\pi_{a,\delta} = \{t_i,\ i = 0, 1, \ldots\}$, for any initial state $x_0 \in \mathcal{C}_R$, the solution corresponding to x_0 and to the sampled-data feedback control law $u(t) = F(x_{t_k})$, $t_k \leq t < t_{(k+1)}$, $k = 0, 1, \ldots,$ exists $\forall t \geq 0$ and, furthermore, satisfies:*

$$x_t \in \mathcal{C}_E,\ \forall t \geq 0; \qquad x_t \in \mathcal{C}_r,\ \forall t \geq T$$

Theorem 9. *(P., SICON 2014) Any steepest descent feedback* k *(continuous or not) stabilizes the system described by (1) in the sample-and-hold sense.*

An Example from Sliding Mode Control

delay-free case studied in Khalil's book.

$$\dot{x}_1(t) = x_2(t),$$
$$\dot{x}_2(t) = H(x_t) + G(x_t)u(t),$$
$$x(\tau) = x_0(\tau), \quad \tau \in [-\Delta, 0],$$

where: $x(t) = [x_1(t) \;\; x_2(t)]^T \in R^2$; Δ is an arbitrary positive real; $H : \mathcal{C} \to R$, $G : \mathcal{C} \to R^+$ are uncertain maps, Lipschitz on bounded sets; $H(0) = 0$; $x_0 \in W^{1,\infty}$, $ess\,\text{sup}_{\theta \in [-\Delta, 0]} \left| \frac{dx_0(\theta)}{d\theta} \right| \leq q$; q is an arbitrary positive constant; $u(t) \in R$ is the control input.

We introduce the following standard assumption.

1) there exists a positive real g_0 such that, for all $\phi \in \mathcal{C}$, the inequality holds $G(\phi) \geq g_0$;

2) there exist a positive real a_1, a locally bounded function $\rho : \mathcal{C} \to R^+$ such that, for all $\phi \in \mathcal{C}$, the inequality holds $|a_1\phi_2(0) + H(\phi)| \leq \rho(\phi)G(\phi)$

Let us consider the Lyapunov-Krasovskii functional $V : \mathcal{C} \to R^+$ defined, for $\phi = [\ \phi_1 \ \ \phi_2\ ]^T \in \mathcal{C}$, as

$$V(\phi) = (a_1\phi_1(0) + \phi_2(0))^2 + \left\{ \begin{array}{cc} \frac{1}{2}\gamma\phi_1^2(0), & |\phi_1(0)| \leq 1, \\ \gamma\left(|\phi_1(0)| - \frac{1}{2}\right), & |\phi_1(0)| > 1, \end{array} \right\},$$

with γ a suitable positive parameter which will be chosen later. Such functional is a CLKF. Indeed, for any $\phi = \left[\begin{array}{cc} \phi_1 & \phi_2 \end{array} \right]^T \in \mathcal{C}$, $u \in R$, by the equality $\phi_1(0)\phi_2(0) = \phi_1(0)(a_1\phi_1(0) + \phi_2(0)) - a_1\phi_1^2(0)$, we have

$$D^+V(\phi, u) \leq 2G(\phi)\left(a_1\phi_1(0) + \phi_2(0)\right)\left(\frac{a_1\phi_2(0) + H(\phi)}{G(\phi)} + u\right)$$
$$+\gamma|a_1\phi_1(0) + \phi_2(0)| - \gamma a_1 \min\left\{|\phi_1(0)|, \phi_1^2(0)\right\}$$

Taking into account of the possibility of choosing (a sliding mode control feedback) $u = -(\rho(\phi) + k_0) \cdot sgn(a_1\phi_1(0) + \phi_2(0))$, with k_0 a positive real, we have, for all $\phi \in \mathcal{C}$,

$$\inf_{u \in R} D^+V(\phi, u) \leq D^+V(\phi, -(\rho(\phi) + k_0) \cdot sgn(a_1\phi_1(0) + \phi_2(0))) \leq$$
$$-(2g_0k_0 - \gamma)\,|a_1\phi_1(0) + \phi_2(0)| - \gamma a_1 \min\left\{|\phi_1(0)|, \phi_1^2(0)\right\}.$$

Therefore, by choosing any $\gamma \in (0, 2g_0k_0)$, it follows that $\inf_{u \in R} D^+V(\phi, u) < 0$, $\forall \phi \in \mathcal{C}$, $\phi(0) \neq 0$. Let the map $k : \mathcal{C} \to R$ be defined, for $\phi \in \mathcal{C}$, as

$$k(\phi) = -(\rho(\phi) + k_0)\, sgn(a_1\phi_1(0) + \phi_2(0))$$

The map k is a steepest descent feedback. Indeed, let $m = \mu = 1$, $s \geq 0$, $\eta = \min\{2g_0 k_0 - \gamma,\ a_1\}$, $p(s) = log_n(1+s)$, $s \geq 0$. We have, for any $\phi \in \mathcal{C}$, taking into account of the increasing property of the function p, and that $V(\phi) = V_1(\phi(0))$,

$$
\begin{aligned}
& D^+V(\phi, k(\phi)) + \eta \max\{0,\ D^+ p \circ V_1(\phi, k(\phi)) + \mu p \circ V_1(\phi(0))\} \leq \\
& - \min\{2g_0 k_0 - \gamma,\ a_1\} \Big(|a_1 \phi_1(0) + \phi_2(0)| + \gamma \min\{|\,\phi_1(0)|,\ \phi_1^2(0)\} \Big) \\
& + \min\{2g_0 k_0 - \gamma,\ a_1\}\, log_n \Big(1 + (a_1 \phi_1(0) + \phi_2(0))^2 \\
& + \gamma \min\Big\{ |\phi_1(0)|,\ \phi_1^2(0) \Big\} \Big)
\end{aligned}
$$

By the inequality $log_n(1 + s_1^2 + s_2) - s_1 - s_2 \leq 0$, $\forall\ s_1, s_2 \in R^+$, it follows that, $\forall \phi \in \mathcal{C}$,

$$
D^+V(\phi, k(\phi)) + \eta \max\{0,\ D^+ p \circ V_1(\phi, k(\phi)) + \mu p \circ V_1(\phi(0))\} \leq 0,
$$

that is, k is a steepest descent feedback.

We conclude that the steepest descent feedback k stabilizes the system in the sample-and-hold sense. The piece-wise constant control law is defined as follows, for $t \geq 0$,

$$u(t) = -\left(\rho\left(x_{t_k}\right) + k_0\right) sgn(a_1 x_1(t_k) + x_2(t_k)),$$
$$t_k \leq t < t_{(k+1)}, \quad k = 0, 1, \ldots, \quad t_0 = 0.$$

Simulations have been performed with H, G defined, for $\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \in \mathcal{C}$, as $H(\phi) = b_1 \phi_1(-\Delta)\phi_2(-\Delta)$, $G(\phi) = b_2$, where b_i, $i = 1,2$ are uncertain parameters, $b_1 \in [-1,1]$, $b_2 \in [1,2]$, Δ is a known positive constant. We can choose, in this case, $a_1 = 1$, $\rho(\phi) = |\phi_1(-\Delta)\phi_2(-\Delta)| + |\phi_2(0)|$, $\phi \in \mathcal{C}$. In the performed simulations, $k_0 = 0.1$, $\Delta = 1.4$, $x_0(\tau) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\tau \in -[\Delta, 0]$, $a = 1$ (uniform sampling), $b_1 = -1$, $b_2 = 1$ are chosen. In simulations, a disturbance $d(t) = d_k$, $k\delta \leq t < (k+1)\delta$, $k = 0, 1, \ldots$, adding to the control law, is also considered. Such disturbance is generated at each sampling time as an element of the interval $[-0.15, 0.05]$ with uniform probability density function.

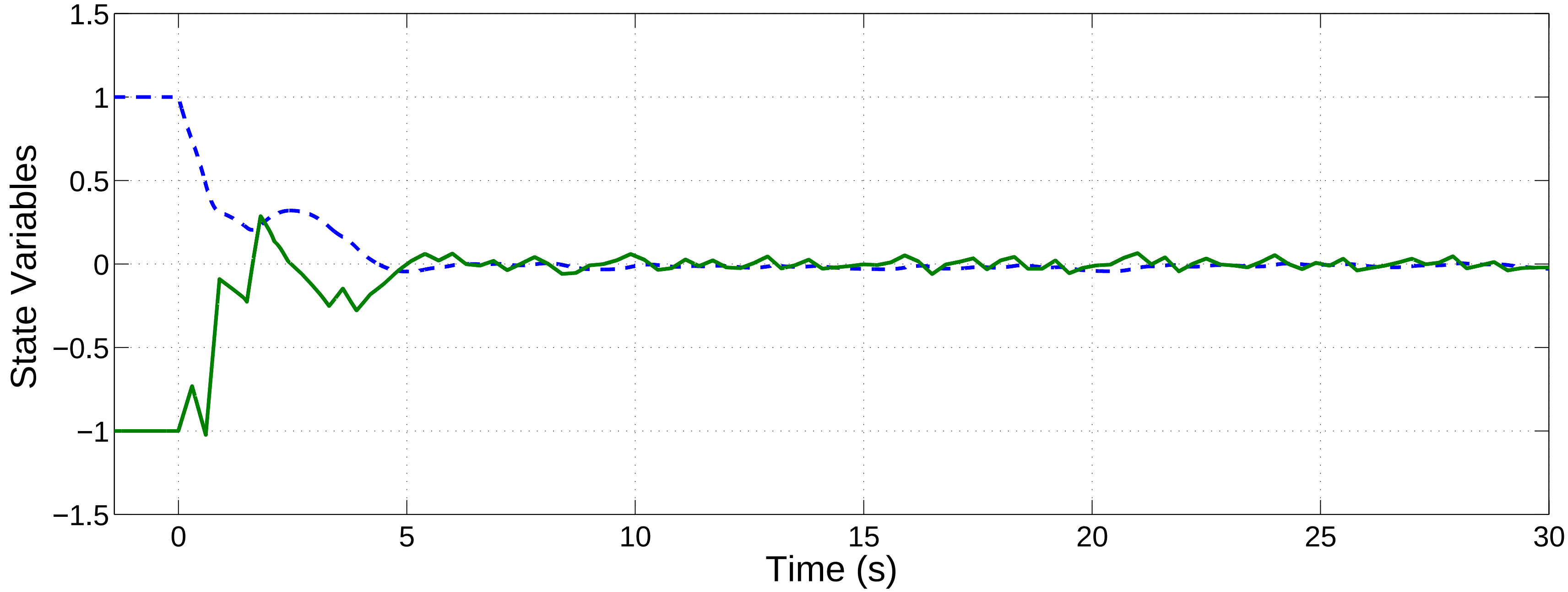


Figure 1: Variables x_1 and x_2, $\delta = 0.3$

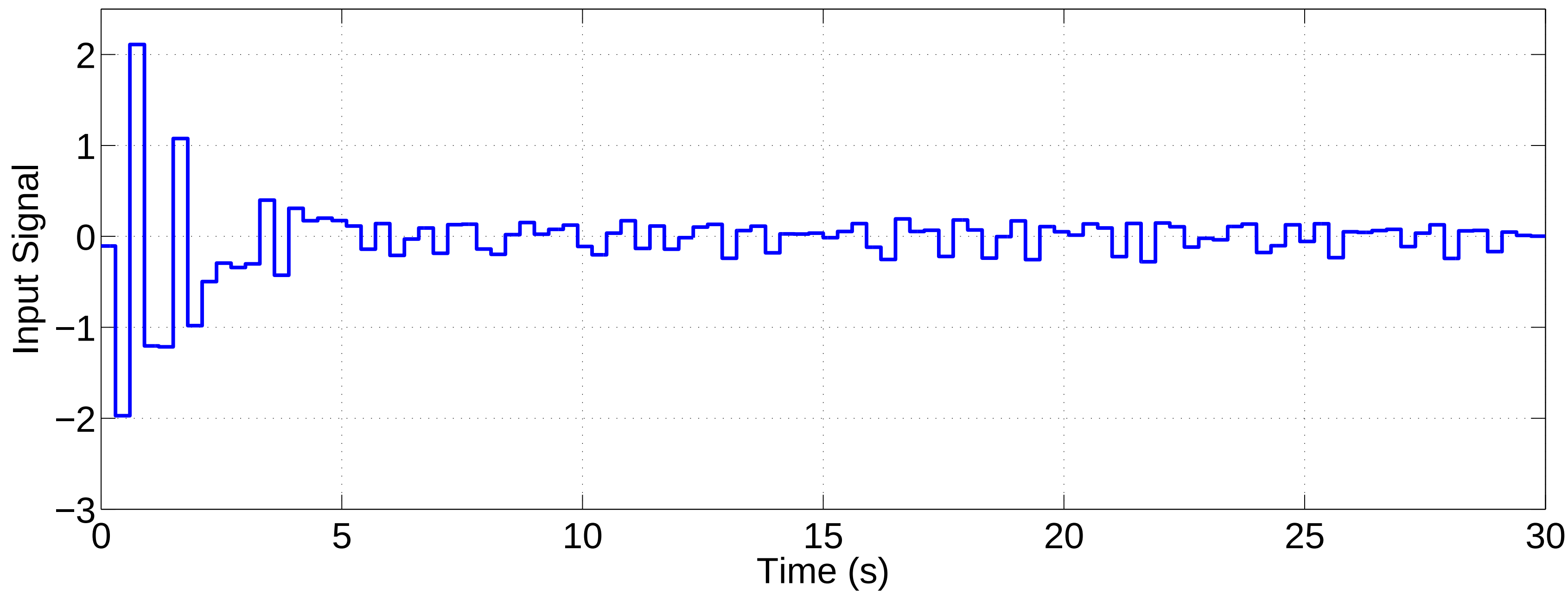


Figure 2: Input Signal (plus disturbance), $\delta = 0.3$

Local Results

Definition 10. *(P., SICON 2014) Let Q be a positive real. We say that a feedback $F : \mathcal{C}_Q \to R^m$ (continuous or not) stabilizes the system described by the RFDE in the sample-and-hold sense, in $\mathcal{C}_Q$, if, for every positive reals r, R, $0 < r < R \leq Q$, $a \in (0,1]$, there exist a positive real δ depending upon r, R, q and Δ, a positive real T, depending upon r, R, q, Δ and a, and a positive real E, depending upon R and Δ, such that, for any partition $\pi_{a,\delta} = \{t_i,\ i = 0, 1, \ldots\}$, for any initial state $x_0 \in \mathcal{C}_R$, the solution corresponding to x_0 and to the sampled-data feedback control law*

$$u(t) = F(x_{t_k}), \quad t_k \leq t < t_{(k+1)}, \quad k = 0, 1, \ldots,$$

exists $\forall t \geq 0$ and, furthermore, satisfies:

$$x_t \in \mathcal{C}_E,\ \forall t \geq 0; \qquad x_t \in \mathcal{C}_r,\ \forall t \geq T$$

Theorem 11. *(P., SICON 2014) Let there exist a positive real S, a functional $V : \mathcal{C}_S \to R^+$, a map $k : \mathcal{C}_S \to R^m$ (continuous or not) such that:*

i) V is a CLKF in $\mathcal{C}_S$;

ii) k is a steepest descent feedback induced by V, in $\mathcal{C}_S$.

Then, the steepest descent feedback k stabilizes the system described by the RFDE in the sample-and-hold sense, in $\mathcal{C}_Q$, where Q is a positive real satisfying the inequality $\alpha_1(S) > \alpha_2(Q)$, with

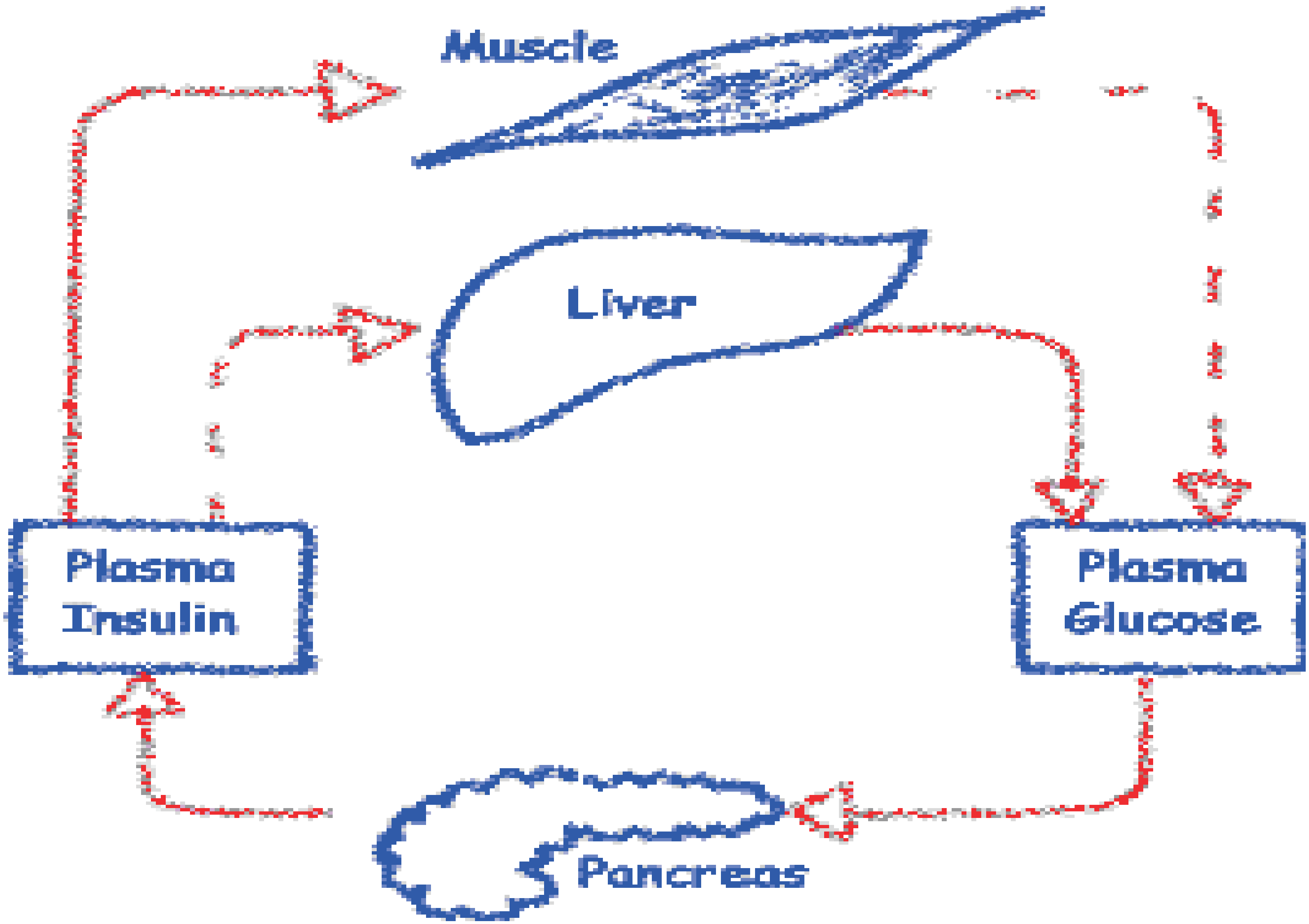
$$\alpha_1(s) = \eta e^{-\mu\Delta} p \circ \beta_1(s), \quad \alpha_2(s) = \gamma_2(s) + \eta p \circ \beta_2(s), \quad s \geq 0.$$

Corollary 12. *Let there exist a diffeomorphism* $\Psi : \Omega_x \to \Omega_z$*, with* $\Omega_x, \Omega_z \in R^n$ *open, bounded neighborhoods of the origin, functions* $\underline{\gamma}_\psi$, $\overline{\gamma}_\psi$*, of class* $\mathcal{K}_\infty$*, a Hurwitz matrix* $F \in R^{n\times n}$*, a positive real* S*, a Lipschitz feedback* $k : \mathcal{C}_S \to R^m$*, zero at zero, such that:* $B_S \subset \Omega_x$*;*

$$\underline{\gamma}_\psi(|x|) \le |\Psi(x)| \le \overline{\gamma}_\psi(|x|), \qquad \forall x \in \Omega_x;$$

$$\left.\frac{\partial \Psi(x)}{\partial x}\right|_{x=\phi(0)} f(\phi, k(\phi)) = F\Psi(\phi(0)), \qquad \forall \phi \in \mathcal{C}_S$$

Then, there exists a positive real Q *such that the feedback* $k : \mathcal{C}_S \to U$ *stabilizes in the sample-and-hold sense, in* $\mathcal{C}_Q$*, the system described by the RFDE.*



Human Glucose-Insulin System. Delays occur because of the reaction time of the pancreas to plasma-glucose variations.

De Gaetano, Palumbo, Panunzi, DCDS-B 2007

$$\frac{dG(t)}{dt} = -K_{xgi}G(t)I(t) + \frac{T_{gh}}{V_G},$$
$$\frac{dI(t)}{dt} = -K_{xi}I(t) + \frac{T_{iGmax}}{V_I}h\Big(G(t-\tau_g)\Big) + v(t),$$
$$G(\tau) = G_0, \qquad I(\tau) = I_0, \qquad \tau \in [-\tau_g, 0], \tag{2}$$

- $G(t)$ $[mM]$ plasma glucose concentration

- $I(t)$ $[pM]$ plasma insulin concentration

The nonlinear map $h(\cdot)$ models the endogenous pancreatic insulin delivery rate as

$$h(G) = \frac{\left(\frac{G}{G^*}\right)^\gamma}{1 + \left(\frac{G}{G^*}\right)^\gamma}, \tag{3}$$

where γ is the progressivity with which the pancreas reacts to circulating glucose concentrations and G^* is the glycemia at which the insulin release is half of its maximal rate. The control input, $v(t)$, is the exogenous intra-venous insulin delivery rate.

Sample-and-hold stabilizer

Let G_{ref} be a positive constant, safe level of glycemia. Let I_{ref} and v_{ref} be the positive reals such that (G_{ref}, I_{ref}) is an equilibrium point for the glucose-insulin system described by the RFDE, forced by the constant input $v(t) = v_{ref}$. The RFDE can be rewritten with the new variables $x(t) = \begin{bmatrix} G(t) - G_{ref} \\ I(t) - I_{ref} \end{bmatrix}$ and with the new input $u(t) = v(t) - v_{ref}$. Let $\overline{\Psi} : R^2 \to R^2$ be defined, for $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in R^2$, as

$$\overline{\Psi}(x) = \begin{bmatrix} x_1 \\ -K_{xgi}(x_1 + G_{ref})(x_2 + I_{ref}) + K_{xgi} G_{ref} I_{ref} \end{bmatrix}. \quad (4)$$

Let Ω_x, Ω_z be open neighborhoods of the origin such that the map $\Psi : \Omega_x \to \Omega_z$ defined, for $x \in \Omega_x$, as $\Psi(x) = \overline{\Psi}(x)$, is a diffeomorphism.

Let us consider the state feedback $k : \mathcal{C} \to R$ defined, for $\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \in \mathcal{C}$, as (Palumbo, P., Panunzi, De Gaetano, DCDS-B 2009)

$$k(\phi) = \begin{cases} -v_{ref} + \frac{\mathcal{P}\Big(\phi_1(0)+G_{ref}, \phi_2(0)+I_{ref}, \phi_1(-\tau_g)+G_{ref}\Big) - R\Psi(\phi(0))}{K_{xgi}(\phi_1(0)+G_{ref})}, \\ \qquad \phi_1(0) \neq -G_{ref}, \\ \\ -v_{ref}, \qquad \phi_1(0) = -G_{ref}, \end{cases} \tag{5}$$

where $\mathcal{P} : R^3 \to R$ is defined, for $y = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T \in R^3$ as

$$\begin{aligned} \mathcal{P}\Big(y_1, y_2, y_3\Big) = & -K_{xgi} y_2 \left(-K_{xgi} y_1 y_2 + \frac{T_{gh}}{V_G} \right) \\ & -K_{xgi} y_1 \left(-K_{xi} y_2 + \frac{T_{iGmax}}{V_I} h\Big(y_3\Big) \right) \end{aligned} \tag{6}$$

Since the exogenous intra-venous insulin delivery rate cannot be negative, we have in this case that the input $v(t)$ must belong to the following set $\overline{V} = [0, v_{max}]$, where v_{max} is a suitable positive real. It follows that $u(t)$ (and thus $k(\phi)$) must belong to the set $U = [-v_{ref}, v_{max} - v_{ref}]$. Since $\frac{\mathcal{P}(G_{ref}, I_{ref}, G_{ref})}{k_{xgi} G_{ref}} = v_{ref} > 0$, taking into account that $\Psi(0) = 0$, it follows that there exists a positive real S such that, for all $\phi \in \mathcal{C}_S$, $k(\phi) \in U$.

Digital Implementation of Glucose Controller

Theorem 13. *(Palumbo, P., Panunzi, De Gaetano, III DelSys Workshop, Grenoble, 2014)*

There exists a positive real Q such that the state feedback k stabilizes in the sample-and-hold sense, in $\mathcal{C}_Q$, the glucose insulin-system.

The piece-wise constant control law $v(t)$ for the glucose-insulin system is defined as follows, for $t \geq 0$,

$$v(t) = \frac{\mathcal{P}\Big(G(t_k), I(t_k), G(t_k - \tau_g)\Big) - R\Psi\left(\begin{bmatrix} G(t_k) - G_{ref} \\ -K_{xgi}G(t_k)I(t_k) + \frac{Tgh}{V_G} \end{bmatrix}\right)}{K_{xgi}G(t_k)},$$

$$t_k \leq t \leq t_{(k+1)}, \;\; k = 0, 1, \ldots, \;\; t_0 = 0 \qquad (7)$$

A case of severe hyperglycemia (establishment of a state of frank Type 2 Diabetes Mellitus) is considered in Palumbo, P., Panunzi, De Gaetano, DCDS 2009. The delay τ_g is equal to 24 min.

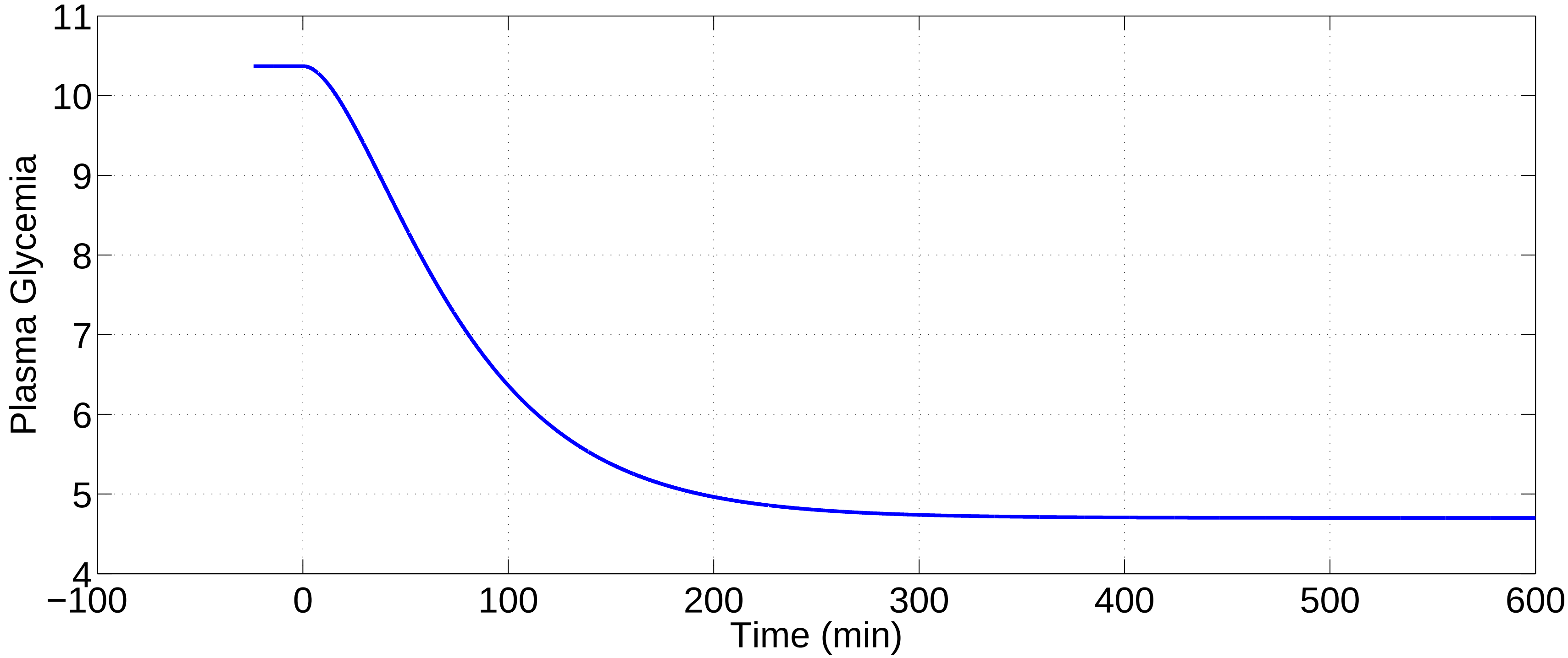


Figure 3: Evolution of the plasma glycemia $G(t)$, with sampling period $\delta = 5\ min$

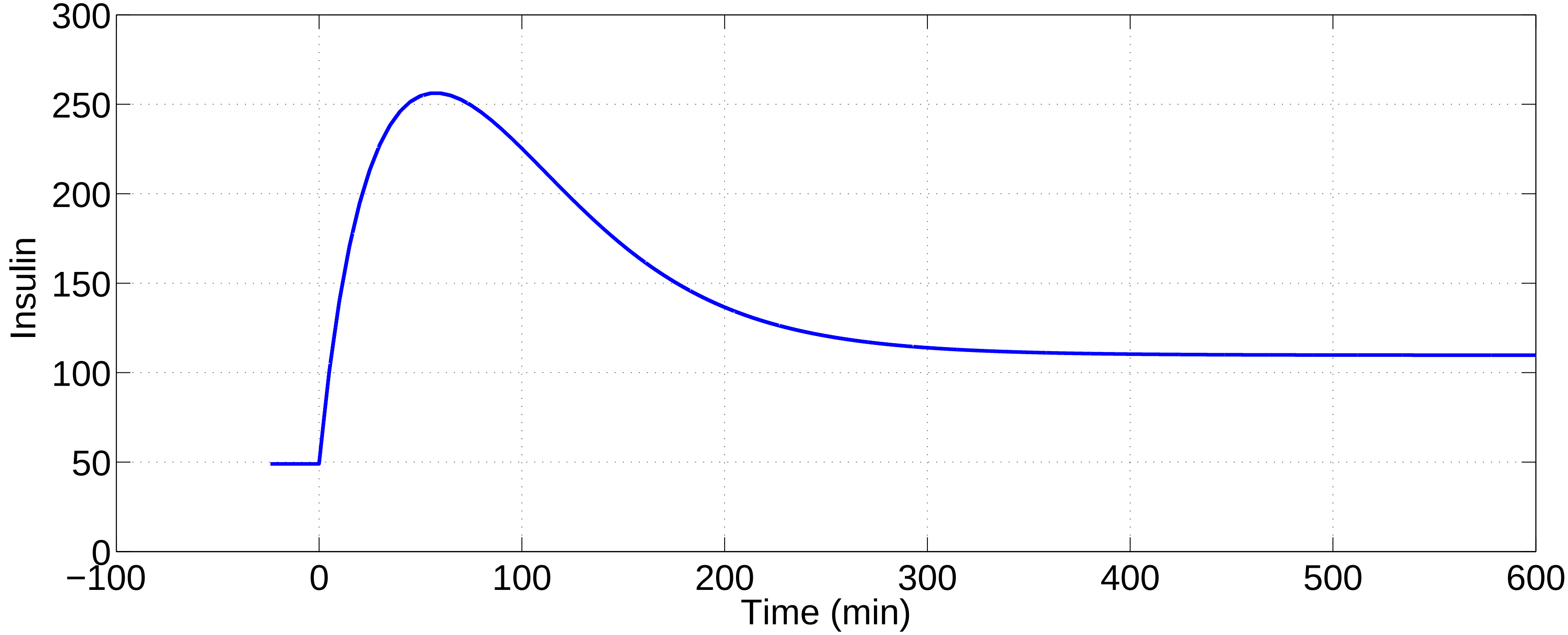


Figure 4: Evolution of the insulin $I(t)$, with sampling period $\delta = 5\ min$

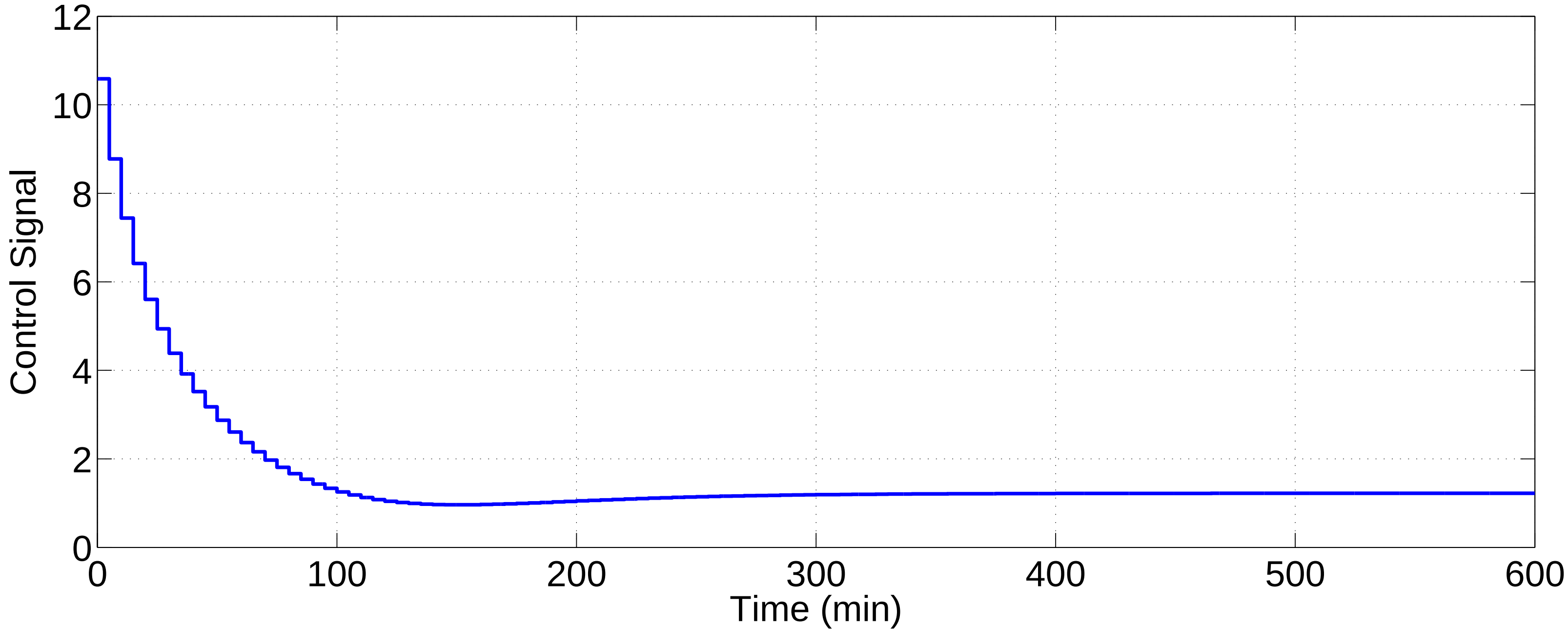


Figure 5: Control Signal, with sampling period $\delta = 5\ min$

Linearizing Virtual Stabilizers

Definition 14. *(P., CDC 2015) We say that a locally bounded state feedback $G : \mathcal{C} \to R^m$ (continuous or not) is a linearizing virtual stabilizer for the nonlinear RFDE system, if there exist a non-negative integer ω, non-negative reals Δ_j, $j = 0, 1, \ldots, \omega$, with $0 = \Delta_0 < \Delta_1 < \cdots < \Delta_\omega = \Delta$, matrices $A_j \in R^{n \times n}$, $j = 0, 1, \ldots, \omega$, such that, for any $\phi \in \mathcal{C}$, the equality holds, for the map f describing the dynamics of the nonlinear RFDE system,*

$$f(\phi, G(\phi)) = \sum_{j=0}^{\omega} A_j \phi(-\Delta_j),$$

and the linear time-delay system described by the equation

$$\dot{\xi}(t) = \sum_{j=0}^{\omega} A_j \xi(t - \Delta_j), \qquad \xi_0 \in \mathcal{C},$$

is 0-GAS.

$\dot{x}(t) = -x(t) + 2x(t - \Delta) + |x(t)|u(t)$

In this case, the map f is defined, for $\phi \in \mathcal{C}$, $u \in R$, as $f(\phi, u) = -\phi(0) + 2\phi(-\Delta) + |\phi(0)|u$. The discontinuous map G defined, for $\phi \in \mathcal{C}$, as $G(\phi) = -2sgn(\phi(0))$, is a linearizing virtual stabilizer. It seems hard (maybe impossible) to find out a continuous map such that the same goal of linearization and (virtual) stabilization is achieved.

Corresponding linear system:

$$\dot{\xi}(t) = -3\xi(t) + 2\xi(t - \Delta) \qquad (8)$$

Lemma 15. *(Kharitonov, Zhabko, AUT 2003) If* $k : \mathcal{C} \longrightarrow R^m$ *is a linearizing virtual stabilizer, then there exists a function* $U : [-\Delta, \Delta] \rightarrow R^{n\times n}$ *with the following properties:*

i) $U(0)$ *is a symmetric positive definite matrix in* $R^{n\times n}$*;*

ii) U *is continuous in* $[-\Delta, \Delta]$ *and continuously differentiable in* $[-\Delta, 0) \cup (0, \Delta]$*, with*

$$\lim_{\tau\to 0^+} \frac{dU(\tau)}{d\tau} = \lim_{\tau\to 0^-} \frac{dU(\tau)}{d\tau} - I_n; \qquad (9)$$

iii) for the functional $W_U : \mathcal{C} \to R^+$ *defined, for* $\phi \in \mathcal{C}$*, as*

$$W_U(\phi) = \phi^T(0)U(0)\phi(0) + 2\phi^T(0) \sum_{j=1}^{\omega} \int_{-\Delta_j}^{0} U(-\theta - \Delta_j)A_j\phi(\theta)d\theta$$
$$+ \sum_{i=1}^{\omega} \sum_{j=1}^{\omega} \int_{-\Delta_i}^{0} \phi^T(\theta_1)A_i^T \left(\int_{-\Delta_j}^{0} U(\theta_1 + \Delta_i - \theta_2 - \Delta_j)A_j\phi(\theta_2)d\theta_2 \right) d\theta_1$$
$$+ \sum_{j=1}^{\omega} \int_{-\Delta_j}^{0} (1 + \Delta_j + \theta)\phi^T(\theta)\phi(\theta)d\theta$$

the following inequalities hold, for suitable positive reals a_i*,* $i = 1, 2,$

$$a_1|\phi(0)|^2 \leq W_U(\phi) \leq a_2\|\phi\|_\infty^2,$$

$$D^+W_U(\phi, k(\phi)) \leq -\left| \begin{bmatrix} \phi^T(0) & \phi^T(-\Delta_1) & \dots & \phi^T(-\Delta_\omega) \end{bmatrix}^T \right|^2$$

Theorem 16. *(P., CDC 2015) Any linearizing virtual stabilizer $k : \mathcal{C} \to R^m$ is a stabilizer in the sample-and-hold sense.*

The linear case

$$\dot{x}(t) = \sum_{j=1}^{p} A_j x(t - \Delta_j) + Bu(t), \qquad x_0 \in \mathcal{C}, \tag{10}$$

Corollary 17. *(P., CDC 2015) Let there exist* $(p+1)$ *matrices* $K_j \in R^{m\times n}$, $j = 0, 1, \ldots, p$, *such that the closed-loop system with*

$$u(t) = K \left[\begin{array}{cccc} x^T(t) & x^T(t-\Delta_1) & \cdots & x^T(t-\Delta_p) \end{array} \right]^T,$$
$$K = \left[\begin{array}{cccc} K_0 & K_1 & \ldots & K_p \end{array} \right],$$

is 0*-GAS. Then, the feedback* $k : \mathcal{C} \to R^m$, *defined, for* $\phi \in \mathcal{C}$, *as* $k(\phi) = K \left[\begin{array}{cccc} \phi^T(0) & \phi^T(-\Delta_1) & \cdots & \phi^T(-\Delta_p) \end{array} \right]^T$, *is a stabilizer in the sample-and-hold sense for the linear system.*

Work in Progress and Future Developments

- Sampled-data observer-based (continuous time) controllers for systems described by RFDEs.

- Sample-and-hold stabilizers for nonlinear systems with time-varying time-delays.

- Stabilization in the sample-and-hold sense of systems described by RFDEs with discontinuous right-hand side.

- Robustness with respect to actuation disturbances and observation errors.

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